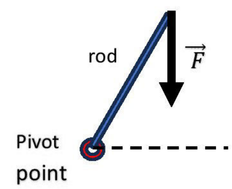
MATH1336 – 01/02 Midterm 2 Review – Part 2 Winter 2025

Part 1 is the review for Midterm 1. Review both! Remember, this exam is comprehensive. It will be approximately 40% on Part 1 and 60% on Part 2.

1. a) Determine the work done by a 20 Newton force directed $\frac{\pi}{4}$ radians above horizontal in pulling a sled 5 meters up a slope measuring $\frac{\pi}{6}$ radians above horizontal.

   b) Determine the torque on a 4-meter rod attached at the origin (pivot point in the diagram) sloping at an angle of $\frac{\pi}{3}$ radians above horizontal by a force of 10 Newtons pulling on the end of the rod away from the origin straight down in the vertical direction.



2. a) Give the equation of the line through: $P_1 = (3, 1, -2)$ and $P_2 = (5, 0, 1)$ in these three forms: $\overrightarrow{PQ} = t\vec{v}$; Parametric form; and Symmetric form. $P = \text{either } P_1 \text{ or } P_2$ and $Q = (x, y, z)$

   b) Determine the distance from the point $M = (5, 4, 2)$ to the line.

   c) Determine the area of the triangle $\Delta P_1 P_2 M$

3. a) Give the equation of the plane containing the given points, $P_1 = (2, 4, -1)$, $P_2 = (6, 2, 3)$, and $P_3 = (4, 0, 7)$, in vector, scalar, and general form.

   b) Determine the distance from the point $P = (8, 2, -4)$ to the plane

4. a) Given the 2-D vector-valued function, $\vec{d}(t) = \langle 4t - t^2, 6t - 2t^2 \rangle$ for the displacement of an object from the origin, graph $\vec{d}$(t) for $t \in \{0, 1, 2, 3, 4, 5\}$

   b) Find $\vec{v}(\text{t}) = \vec{d}'(t)$. Graph $\vec{v}$(t) for $t \in \{0, 1, 2, 3, 4, 5\}$ How do the directions of your $\vec{v}$(t) vectors correspond to your graph of $\vec{d}$(t) ?

   c) Find $\vec{a}(\text{t}) = \vec{v}'(t)$. What does your answer imply?

5. Given: $\vec{v}(\text{t}) = \langle 4\, e^{\left(-\frac{t}{4}\right)} \cos\left(\frac{\pi}{4} t\right), 2\, e^{\left(-\frac{t}{4}\right)} \sin\left(\frac{\pi}{4} t\right), \frac{3t}{t+2} \rangle$

   a) Determine the limit: *i)* $\lim\limits_{t \to 0} \vec{v}(\text{t})$ *ii)* $\lim\limits_{t \to 4} \vec{v}(\text{t})$ *iii)* $\lim\limits_{t \to \infty} \vec{v}(\text{t})$

   b) Describe the graph.

6. Write the formula and the recursive definition of each sequence:

   a) $\{a_\text{n}\} = \{5, 10, 20, 40, 80, \dots\}$

   b) $\{b_\text{n}\} = \{5, 8, 11, 14, 17, 20, \dots\}$

   c) $\{c_\text{n}\} = \{1, 3, 7, 15, 31, 63, \dots\}$ recursive only

   d) $\{d_\text{n}\} = \left\{2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \dots\right\}$

7. Make an educated/informed "guess" of the limit of the sequence $a_n = \frac{4\text{n}}{2\text{n}-1}$, *i.e.*, $L = \lim\limits_{n \to \infty} a_n$

   a) Find $M \in \mathbb{N}$ (in terms of ε) such that $|a_n - L| < \varepsilon \ \forall n > M$ Doing this proves $\lim\limits_{n \to \infty} a_n = L$

   b) Find $M \in \mathbb{N}$ such that $|a_n - L| < \varepsilon \ \forall n > M$ for $\varepsilon = 0.1$ and $\varepsilon = 0.01$

8. Make an educated/informed "guess" of the limit of the sequence $a_n = \frac{2\text{n}}{\text{n}^2+1}$, *i.e.*, $L = \lim_{n\to\infty} a_n$

   a) Find $M \in \mathbb{N}$ (in terms of ε) such that $|a_n - L| < \varepsilon \ \forall n > M$ Doing this proves $\lim_{n\to\infty} a_n = L$

   b) Find $M \in \mathbb{N}$ such that $|a_n - L| < \varepsilon \ \forall n > M$ for $\varepsilon = 0.1$ and $\varepsilon = 0.01$

9. a) If an infinite series, $\sum_{i=1}^{\infty} a_i$ converges, what can you say with certainty about the limit $L = \lim_{n\to\infty} a_n$? Write your answer as an if/then statement, e.g., If A then B.

   b) Is the converse of the statement in part a) above true? If not, provide an explanation or counterexample supporting your answer. The converse of "If A then B" is "If B then A."

10. For each of the following series, apply Theorem 5.8, the "Divergence Test." Make sure to state the conclusion accurately.

    a) $\sum_{i=1}^{\infty} \left(\frac{3}{2}\right)^{i-1}$ b) $\sum_{n=1}^{\infty} 2^{-\frac{1}{\text{n}}}$ c) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ d) $\sum_{n=1}^{\infty} \frac{n}{\ln n}$

11. a) Show $\sum_{i=1}^{\text{n}} r^{i-1} = \frac{1-r^n}{1-r}$ b) For what values of r will $\sum_{i=1}^{\infty} \text{r}^{i-1}$ have a limit?

12. a) Find the limit of $\sum_{i=1}^{\infty} 6\left(\frac{2}{3}\right)^{i-1}$ b) Find the limit of $\sum_{i=1}^{\infty} 6\left(-\frac{2}{3}\right)^{i-1}$

    c) Which of the limits in parts a) and b) above is less? Why?

13. a) Find the limit of $\sum_{i=1}^{\infty} 10\left(\frac{2}{5}\right)^{i}$ b) Find the limit of $\sum_{i=3}^{\infty} \left(\frac{1}{3}\right)^{i-1}$

14. For each of the following *p*-series, apply Theorem 5.8, the "Divergence Test." If the "Divergence Test" is inconclusive, apply the "Integral Test" to determine whether the given *p*-series converges or diverges.

    a) $\sum_{i=1}^{\infty} \frac{1}{i^{-0.5}}$ b) $\sum_{i=1}^{\infty} \frac{1}{i^{0.5}}$ c) $\sum_{i=1}^{\infty} \frac{1}{i}$ d) $\sum_{i=1}^{\infty} \frac{1}{i^{1.1}}$

15. Use the "Comparison Test" to determine if the given series converges or diverges.

    a) $\sum_{i=1}^{\infty} \frac{1}{\sqrt{i^3+3i}}$ b) $\sum_{i=1}^{\infty} \left(\frac{i}{3i+1}\right)^{i}$