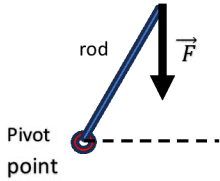


**Part 1 is the review for Midterm 1. Review both! Remember, this exam is comprehensive. It will be approximately 40% on Part 1 and 60% on Part 2.**

1.
    - a) Determine the work done by a 20 Newton force directed  $\frac{\pi}{4}$  radians above horizontal in pulling a sled 5 meters up a slope measuring  $\frac{\pi}{6}$  radians above horizontal.
    - b) Determine the torque on a 4-meter rod attached at the origin (pivot point in the diagram) sloping at an angle of  $\frac{\pi}{3}$  radians above horizontal by a force of 10 Newtons pulling on the end of the rod away from the origin straight down in the vertical direction.
- 
2.
    - a) Give the equation of the line through:  $P_1 = (3, 1, -2)$  and  $P_2 = (5, 0, 1)$  in these three forms:  $\overrightarrow{PQ} = t\vec{v}$ ; Parametric form; and Symmetric form.  $P = \text{either } P_1 \text{ or } P_2$  and  $Q = (x, y, z)$
    - b) Determine the distance from the point  $M = (5, 4, 2)$  to the line.
    - c) Determine the area of the triangle  $\Delta P_1 P_2 M$
  3.
    - a) Give the equation of the plane containing the given points,  $P_1 = (2, 4, -1)$ ,  $P_2 = (6, 2, 3)$ , and  $P_3 = (4, 0, 7)$ , in vector, scalar, and general form.
    - b) Determine the distance from the point  $P = (8, 2, -4)$  to the plane
  4.
    - a) Given the 2-D vector-valued function,  $\vec{d}(t) = \langle 4t - t^2, 6t - 2t^2 \rangle$  for the displacement of an object from the origin, graph  $\vec{d}(t)$  for  $t \in \{0, 1, 2, 3, 4, 5\}$
    - b) Find  $\vec{v}(t) = \vec{d}'(t)$ . Graph  $\vec{v}(t)$  for  $t \in \{0, 1, 2, 3, 4, 5\}$  How do the directions of your  $\vec{v}(t)$  vectors correspond to your graph of  $\vec{d}(t)$ ?
    - c) Find  $\vec{a}(t) = \vec{v}'(t)$ . What does your answer imply?
  5. Given:  $\vec{v}(t) = \langle 4e^{(-\frac{t}{4})} \cos(\frac{\pi}{4}t), 2e^{(-\frac{t}{4})} \sin(\frac{\pi}{4}t), \frac{3t}{t+2} \rangle$ 
    - a) Determine the limit: i)  $\lim_{t \rightarrow 0} \vec{v}(t)$  ii)  $\lim_{t \rightarrow 4} \vec{v}(t)$  iii)  $\lim_{t \rightarrow \infty} \vec{v}(t)$
    - b) Describe the graph.
  6. Write the formula and the recursive definition of each sequence:
    - a)  $\{a_n\} = \{5, 10, 20, 40, 80, \dots\}$
    - b)  $\{b_n\} = \{5, 8, 11, 14, 17, 20, \dots\}$
    - c)  $\{c_n\} = \{1, 3, 7, 15, 31, 63, \dots\}$  recursive only
    - d)  $\{d_n\} = \left\{2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \dots\right\}$
  7. Make an educated/informed “guess” of the limit of the sequence  $a_n = \frac{4n}{2n-1}$ , i.e.,  $L = \lim_{n \rightarrow \infty} a_n$ 
    - a) Find  $M \in \mathbb{N}$  (in terms of  $\varepsilon$ ) such that  $|a_n - L| < \varepsilon \quad \forall n > M$  Doing this proves  $\lim_{n \rightarrow \infty} a_n = L$
    - b) Find  $M \in \mathbb{N}$  such that  $|a_n - L| < \varepsilon \quad \forall n > M$  for  $\varepsilon = 0.1$  and  $\varepsilon = 0.01$

8. Make an educated/informed “guess” of the limit of the sequence  $a_n = \frac{2n}{n^2+1}$ , i.e.,  $L = \lim_{n \rightarrow \infty} a_n$
- a) Find  $M \in \mathbb{N}$  (in terms of  $\varepsilon$ ) such that  $|a_n - L| < \varepsilon \quad \forall n > M$  Doing this proves  $\lim_{n \rightarrow \infty} a_n = L$
- b) Find  $M \in \mathbb{N}$  such that  $|a_n - L| < \varepsilon \quad \forall n > M$  for  $\varepsilon = 0.1$  and  $\varepsilon = 0.01$
9. a) If an infinite series,  $\sum_{i=1}^{\infty} a_i$  converges, what can you say with certainty about the limit  $L = \lim_{n \rightarrow \infty} a_n$ ?  
Write your answer as an if/then statement, e.g., If A then B.
- b) Is the converse of the statement in part a) above true? If not, provide an explanation or counterexample supporting your answer. The converse of “If A then B” is “If B then A.”
10. For each of the following series, apply Theorem 5.8, the “Divergence Test.” Make sure to state the conclusion accurately.
- a)  $\sum_{i=1}^{\infty} \left(\frac{3}{2}\right)^{i-1}$       b)  $\sum_{n=1}^{\infty} 2^{-\frac{1}{n}}$       c)  $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$       d)  $\sum_{n=1}^{\infty} \frac{n}{\ln n}$
11. a) Show  $\sum_{i=1}^n r^{i-1} = \frac{1-r^n}{1-r}$       b) For what values of  $r$  will  $\sum_{i=1}^{\infty} r^{i-1}$  have a limit?
12. a) Find the limit of  $\sum_{i=1}^{\infty} 6 \left(\frac{2}{3}\right)^{i-1}$       b) Find the limit of  $\sum_{i=1}^{\infty} 6 \left(-\frac{2}{3}\right)^{i-1}$
- c) Which of the limits in parts a) and b) above is less? Why?
13. a) Find the limit of  $\sum_{i=1}^{\infty} 10 \left(\frac{2}{5}\right)^i$       b) Find the limit of  $\sum_{i=3}^{\infty} \left(\frac{1}{3}\right)^{i-1}$
14. For each of the following  $p$ -series, apply Theorem 5.8, the “Divergence Test.” If the “Divergence Test” is inconclusive, apply the “Integral Test” to determine whether the given  $p$ -series converges or diverges.
- a)  $\sum_{i=1}^{\infty} \frac{1}{i^{-0.5}}$       b)  $\sum_{i=1}^{\infty} \frac{1}{i^{0.5}}$       c)  $\sum_{i=1}^{\infty} \frac{1}{i}$       d)  $\sum_{i=1}^{\infty} \frac{1}{i^{1.1}}$
15. Use the “Comparison Test” to determine if the given series converges or diverges.
- a)  $\sum_{i=1}^{\infty} \frac{1}{\sqrt{i^3+3i}}$       b)  $\sum_{i=1}^{\infty} \left(\frac{i}{3i+1}\right)^i$