The law above can be derived by classical thermodynamic considerations but not the numerical value of the constant $\sigma$.

- Wien's displacement law (1894). The experimental plots of $u_\lambda(T)$ as a function of $\lambda$ for different temperatures are of the form shown in Figure 1.2. The plot for a given $T$ exhibits a maxima at the wavelength, say $\lambda_m$. Wien noted that the value of the product $\lambda_m T$ is same for the plots at different temperatures:

$$\lambda_m T \approx 0.2898 \text{ cm} \cdot \text{K}. \tag{1.2}$$

This is known as Wien's displacement law.

**Ex. 1.1.** Show that if $\lambda_m T$ = constant, and the intensity is proportional to $T^4$ then $u_\lambda(T)$ is of the form

$$u_\lambda(T) = \frac{1}{\lambda^5} f(\lambda T). \tag{1.3}$$

Hint: The extremum in $u_\lambda(T)$ as the function of $\lambda$ occurs at that value $\lambda_m$ of $\lambda$ for which $\partial u_\lambda(T)/\partial\lambda = 0$. The condition $\lambda_m T$ = constant can be satisfied if $u_\lambda(T) = \lambda^k f(\lambda T)$ since in that case $\partial u_\lambda(T)/\partial\lambda = \lambda^{k-1}\left(kf(x) + x\partial f(x)/\partial x\right)$, $x = \lambda T$. Use this form of $u_\lambda(T)$ in the integral in (1.1) to show that it will be proportional to $T^4$ if $k = -5$.

The laws above are based on experimental observations and not on consideration of the physical processes responsible for black-body emission. The following two laws attempt to construct $u_\lambda(T)$ based on theoretical considerations.

- Wien's law (1896). Wien assumed that the black-body radiation is caused by emission from the moving molecules and that the wavelength of the radiation emitted by a molecule is a function of its speed (Wu, 1986). He assumed also that the Maxwell-Boltzmann law for velocity distribution of a gas of molecules at a constant temperature is applicable also to the molecules in the walls of the cavity. Recall that, according to the law in question, the probability $f(\mathbf{p})$ for a molecule in a gas of molecules, each of mass $m$ in thermal equilibrium at temperature $T$, to have momentum $\mathbf{p}$ is

$$f(\mathbf{p}) = A\exp\left(-|\mathbf{p}|^2/2m\mathrm{k_B}T\right), \tag{1.4}$$

where $\mathrm{k_B}$ is the Boltzmann's constant. Hence, according to Wien, the probability that the molecules emit radiation at the wavelength $\lambda$ is proportional to $\exp\left\{-mv^2/2\mathrm{k_B}T\right\}$. Now, in order that the resulting distribution function for the wavelengths be of the form (1.3), $v^2$ must be proportional to $1/\lambda$ so that

$$u_\lambda(T) = \frac{A}{\lambda^5}\exp\left(-\alpha/\lambda T\right), \qquad \alpha > 0, \tag{1.5}$$

where $A$ and $\alpha$ are the constants to be determined by comparing the expression above with experiments. The assumptions on which this law is based are now known to be erroneous. The law itself, however, is found to be in very good agreement with the short wavelength region of experimental plots of $u_\lambda(T)$ as a function of $\lambda$ for all $T$.

- Rayleigh-Jeans law (1900 and 1905). Rayleigh in 1900 and Jeans in 1905 derived the black-body radiation law based on the premise that the e.m. field is describable as a collection of harmonic oscillators with each mode of the field behaving like an oscillator. Rayleigh therefore considered the radiation inside a cavity at temperature $T$ as a collection of harmonic oscillators in thermal equilibrium with the walls of the cavity. Hence, the energy density of the e.m. field in the cavity